

* 0000800000001 *



**Cambridge Assessment
International Education**

Cambridge International AS & A Level

CANDIDATE
NAME



CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



1 The matrix \mathbf{M} represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor k ($k \neq 0$), followed by a shear, x -axis fixed, with $(0, 1)$ mapped to $(k, 1)$.

(a) Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

(b) The transformation represented by \mathbf{M} has a line of invariant points.

Find, in terms of k , the equation of this line.

[3]





The unit square S in the x - y plane is transformed by \mathbf{M} onto the parallelogram P .



(c) Find, in terms of k , a matrix which transforms P onto S .

[1]

(d) Given that the area of P is $3k^2$ units 2 , find the possible values of k .

[2]





2 Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n}{dx^n}(\tan^{-1}x) = P_n(x)(1+x^2)^{-n},$$

where $P_n(x)$ is a polynomial of degree $n-1$.

[6]

DO NOT WRITE IN THIS MARGIN



* 0000800000005 *



5



DO NOT WRITE IN THIS MARGIN





3 The quartic equation $x^4 + 2x^3 - 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\alpha^4, \beta^4, \gamma^4, \delta^4$ and state the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [5]





(c) Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$.





4 (a) Use the method of differences to find $\sum_{r=1}^n \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n and k , where k is a positive constant. [4]





It is given that $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

(b) Find the value of k . [2]

(c) Hence find $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n . [2]





5 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 6xy$$

has polar equation $r^2 = 3 \sin 2\theta$.

[2]

.....
.....
.....
.....
.....
.....
.....
.....

The curve C has polar equation $r^2 = 3 \sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole.

[3]

DO NOT WRITE IN THIS MARGIN





(c) Find the area of the region enclosed by C .

(d) Find the maximum distance of a point on C from the initial line.

[6]





6 The curve C has equation $y = \frac{4x^2 + x + 1}{2x^2 - 7x + 3}$.

(a) Find the equations of the asymptotes of C .

[2]

(b) Find the coordinates of any stationary points on C .

[4]





(c) Sketch C , stating the coordinates of any intersections with the axes.

.....

(d) Sketch the curve with equation $y = \left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right|$ and state the set of values of k for which $\left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right| = k$ has 4 distinct real solutions. [2]





7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2 .

(a) Find the equation of Π_1 , giving your answer in the form $ax + by + cz = d$. [4]

The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2 . [4]





The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

(c) Find a vector equation for PQ .

[7]





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

