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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/11**

Paper 1 Further Pure Mathematics 1 **October/November 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 The matrix **M** represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor k ($k \neq 0$), followed by a shear, x-axis fixed, with (0, 1) mapped to (k, 1).

(a) Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

(b) The transformation represented by **M** has a line of invariant points.

Find, in terms of k, the equation of this line. [3]

The unit square S in the x-y plane is transformed by $\mathbf{M}$ onto the parallelogram P.

(c) Find, in terms of k, a matrix which transforms P onto S. [1]

(d) Given that the area of P is $3k^2$ units^2, find the possible values of k. [2]

2 Prove by mathematical induction that, for all positive integers n,

$$\frac{d^n}{dx^n}\left(\tan^{-1}x\right) = P_n(x)\left(1+x^2\right)^{-n},$$

where $P_n(x)$ is a polynomial of degree $n-1$. [6]

3 The quartic equation $x^4 + 2x^3 - 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\alpha^4, \beta^4, \gamma^4, \delta^4$ and state the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [5]

(b) Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$. [3]

(c) Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$. [2]

4 **(a)** Use the method of differences to find $\sum_{r=1}^{n} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n and k, where k is a positive constant. [4]

It is given that $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

(b) Find the value of k. [2]

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(c) Hence find $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n. [2]

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5 **(a)** Show that the curve with Cartesian equation

$$\left(x^2+y^2\right)^2 = 6xy$$

has polar equation $r^2 = 3\sin 2\theta$. [2]

The curve C has polar equation $r^2 = 3\sin 2\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

(c) Find the area of the region enclosed by C. [2]

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(d) Find the maximum distance of a point on C from the initial line. [6]

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6 The curve C has equation $y = \dfrac{4x^2 + x + 1}{2x^2 - 7x + 3}$.

(a) Find the equations of the asymptotes of C. [2]

(b) Find the coordinates of any stationary points on C. [4]

(c) Sketch C, stating the coordinates of any intersections with the axes. [5]

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(d) Sketch the curve with equation $y = \left|\dfrac{4x^2 + x + 1}{2x^2 - 7x + 3}\right|$ and state the set of values of k for which $\left|\dfrac{4x^2 + x + 1}{2x^2 - 7x + 3}\right| = k$ has 4 distinct real solutions. [2]

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7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2.

(a) Find the equation of Π_1, giving your answer in the form $ax + by + cz = d$. [4]

The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2. [4]

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(c) Find a vector equation for PQ. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.